region I causes a stress-optical effect that is visible under the petrographic microscope. If a polished thick or thin section of the assemblage is made parallel to a section of zero birefringence in the host away from the influence of the inclusion, this effect, due to a field of deviatoric stress, expresses itself between crossed polars as a piezobirefringent halo. The boundary is then recognizable as the limiting *P-T* line for disappearance of the piezobirefringent halo.

Although it is more nearly precise to consider the assemblage in three dimensions, it is also useful in many cases to reduce the conceptual model to one that is approximately two dimensional rather than three dimensional (Rosenfeld, 1969, p. 318–329, 338). This particularly applies to thin sections in which a host-inclusion combination is cut so thin and in such an orientation as to have the cavity boundary penetrate the section essentially perpendicularly. The same analysis as in the previous paragraph applies, except that only strains parallel to the section are considered. The comparable subdivisions of the *P-T* diagram in this case will, in general, be different from that for the three-dimensional case except for highly symmetric combinations in which both host and inclusion are cubic. The boundaries, in either case, include P_f and T_f.

The principal advantages of the two-dimensional simplification over the three-dimensional approach are: (1) ease of laboratory observation: thin sections readily permit laboratory identification of conditions of equal strains in host and inclusion; (2) the fact that the direction of equal strains is very nearly restricted to the plane of the section; (3) the fact that sectioning permits access of a fluid medium of low viscosity and thus the operating pressure of the system to the region between host and inclusion;[4] and (4) the fact that the piezobirefringent halo becomes sensitive not only to P and T but also to the angular relations among the host, inclusion, and thin section.[5]

As defined in Part I, an *isomeke* is any *P-T* curve along which the distance between two reference points embedded in the hydrostatically stressed host mineral remains equal to that between two reference points embedded in the hydrostatically stressed inclusion mineral. The boundary between regions *I* and *II*, mentioned above, is an isomeke.

Determination of any isomeke, passing through P_f, T_f, is a two-step procedure. First, a comparison dilatometer (see Part I) is constructed using pieces of the host and inclusion minerals being studied. This device is designed to monitor a *set* of isomekes passing through a *P-T* region that is sufficiently large to include any possible values of P_f and T_f. Each isomeke, by definition, shares the property that it is a *P-T* trajectory for which the length of an appropriately oriented line segment between two points of reference[6] embedded in a piece of the inclusion mineral equals that between similarly defined points in a piece of the host mineral. Secondly, let some laboratory pressure (P_n) and temperature (T_n) be found for which the piezobirefringent halo around the inclusion in the host is observed to vanish. The isomeke passing through P_f,T_f is then that particular isomeke passing through P_n,T_n. The intersection of that isomeke with another, independently determined for some other host-inclusion combination in the specimen, uniquely specifies P_f,T_f, assuming that both combinations formed under the same conditions.

Relationship of Solid Inclusion Piezothermometry to Equations of State

Because equations of state provide an alternate means for the determination of isomekes, it is desirable to clarify their interrelationship.

Let the *natural strain* (Nadai, 1950, p. 73–74) $\bar{\epsilon}$ between two points of reference embedded in a given solid be defined as

$$\bar{\epsilon} \equiv \ln\left(\frac{l}{l_0}\right) = \ln(1 + \epsilon) \qquad (1)$$

where

$l \equiv$ distance between the points at some P and T

o $\equiv$ subscript indicating standard conditions, P = 1 bar and T = 25°C, *not* necessarily on a desired isomeke.

[4] Rosenfeld (1969, p. 320–326, 330) makes applications based on the two-dimensional approach, which entails relaxation by thin-sectioning of some or all of the elastic strain due to deviatoric stress. Determination of the angular relations for elimination of that strain is an essential feature of the method. That this approach can only be approximate when dealing with noncubic minerals results from the necessarily finite thickness of the thin section, the irregular shapes of real inclusions, and the possible presence of shear stresses parallel to the host-inclusion contact.

[5] As an example, let θ be the angle between the section normal and c of a uniaxial inclusion (*e.g.*, quartz) in an optically isotropic host (*e.g.*, garnet). A *P-T* region will exist for which the condition of appearance of a piezobirefringent halo at a particular P and T is $\theta > \theta_c$, where θ_c is the limiting value of θ for which a halo appears (Rosenfeld, 1969, p. 320–323).

[6] The line segments connecting the reference points in each piece also must parallel the crystallographic directions within host and inclusion for which the strains are equal in the conceptual model. While it is simple to identify these directions for minerals of high symmetry (see, for example, Rosenfeld, 1969, p. 318–327), a general procedure that is also applicable to minerals of the least symmetric crystal systems has not yet been developed.

$$\epsilon \equiv \frac{l - l_0}{l_0} \equiv \text{the } \textit{conventional strain}.$$

Then the natural strain difference, δ_{x-y}, between the line segments connecting two reference points embedded in solid x and two reference points embedded in solid y at some arbitrary P and T is

$$\delta_{x-y} \equiv \bar{\epsilon}_x - \bar{\epsilon}_y \tag{2}$$

Now an isomeke, by definition, is a curve in P and T along which two reference points embedded in x remain the same distance apart, l_x, as that between two reference points embedded in y, l_y, or

$$l_x = l_y \tag{3}$$

The location of the isomeke is uniquely determined by the ratio of l_y to l_x at standard conditions, l_{y0}/l_{x0}, which is a constant, not necessarily equal to unity, for a given isomeke. To see this, from Equation (1) we obtain

$$\ln l_x = \bar{\epsilon}_x - \ln l_{x0} \tag{4a}$$

$$\ln l_y = \bar{\epsilon}_y - \ln l_{y0} \tag{4b}$$

Subtracting Equation (4b) from (4a) and using Equations (2) and (3), we find

$$\delta_{x-y} = \ln\left(\frac{l_{y0}}{l_{x0}}\right). \tag{5}$$

δ_{x-y} is thus constant along a given isomeke and can, in principle, be determined at standard conditions by measuring l_{x0} and l_{y0}. From Equation (2) the relationship between an isomeke and the equations of state of two solids is also evident. An isomeke is found to be any curve in P and T along which the difference in natural strain is constant. Thus a good graphical way to see the functional relationship between isomekes and equations of state is a P-T diagram with contours of constant natural strain. The superposition of such diagrams for x and y allows determination of isomekes by a simple procedure. Each isomeke is a contour of constant difference, δ_{x-y}, in natural strain.[7] If, eventually, an accurate way for measuring δ_{x-y} can be incorporated into our system of comparison dilatometry and the equation of state of x is known, it follows from Equation (2) that $\bar{\epsilon}_y$ can be determined all along the isomeke characterized by that δ_{x-y} by subtraction of δ_{x-y} from $\bar{\epsilon}_x$. The equation of state of y could thereby be determined by generation of suitably spaced isomekes.

The differential equation of an isomeke also follows from Equation (2). The total differential of δ_{x-y} is

$$\mathrm{d}\,\delta_{x-y} = \left(\frac{\partial\,\delta_{x-y}}{\partial T}\right)_P \mathrm{d}T + \left(\frac{\partial\,\delta_{x-y}}{\partial P}\right)_T \mathrm{d}P \tag{6}$$

At constant δ_{x-y},

$$m_{x-y} \equiv \left(\frac{\partial T}{\partial P}\right)_{\delta_{x-y}} = -\frac{(\partial\,\delta_{x-y}/\partial P)_T}{(\partial\,\delta_{x-y}/\partial T)_P}$$

$$= \frac{-[(\partial\bar{\epsilon}_x/\partial P)_T - (\partial\bar{\epsilon}_y/\partial P)_T]}{(\partial\bar{\epsilon}_x/\partial T)_P - (\partial\bar{\epsilon}_y/\partial T)_P} = \frac{\beta_x - \beta_y}{\alpha_x - \alpha_y} \tag{7}$$

where β and α are, respectively, the isothermal linear compressibility and isobaric linear coefficient of thermal expansion, both simply defined as partial derivatives of $\bar{\epsilon}$.

There is ample reason, therefore, to use $\bar{\epsilon}$-P-T diagrams in analyzing data from comparison dilatometry.

Calibration for Association Quartz-Garnet

The previous calculation of families of isomekes (Rosenfeld, 1969, p. 327–334; there called "integral null curves" or "isogons") for almandine–quartz was based on the very scant data on α's and β's and on long-range extrapolation using more-or-less reasonable boundary conditions.

We present here the experimental calibration of the association quartz (q)–garnet (g), emphasizing garnet of the almandine type (garnet #1, in Table 1 and Fig. 1). Limited experiments on other garnets, whose compositions are also shown in Table 1 and Figure 1, allow some generalization about the effect of solid solution on the almandine-quartz isomekes. All garnets were analyzed by electron microprobe.

Experimental Results

Results for Pair: Quartz–Almandine-Type Garnet

Figures 2, 3, and 4 show data points within the region bounded by the low-high quartz transition and 7 kbar for isomekes of almandine-type garnet (garnet #1) relative to quartz. In Figure 2 the quartz rod is oriented $\perp c$; in Figure 3 it is oriented at an angle of 45° relative to c; and in Figure 4 it is oriented $\parallel c$. The curves in these figures are derived as described in a section below. For convenience in petrographic utilization (Rosenfeld, 1969, p. 327–328), the curves are identified by $\sin^2\theta$, where θ is the angle between

[7] The method of plotting isomekes is identical to that used by geologists in plotting convergence maps, maps showing contours of constant vertical component of thickness of a stratigraphic unit (Lahee, 1941, p. 649–654).